Part 10 - Life Insurance, Mathematical Formulae and Scottish Widows

Life Insurance.

In the ancient world there was not always room for the sick, suffering, disabled, aged, or the poor—these were often not part of the cultural consciousness of societies. Early methods of protection, aside from the normal support of the extended family, involved charity; religious organizations or neighbours who would collect for the destitute and needy. By the middle of the 3rd century, 1,500 suffering people were being supported by charitable operations in Rome. Charitable protection remains an active form of support in the modern era, but receiving charity is uncertain and is often accompanied by social stigma.

Elementary mutual aid agreements and pensions did arise in antiquity. Early in the Roman Empire, associations were formed to meet the expenses of burial, cremation, and monuments—precursors to burial insurance and Friendly Societies. A small sum was paid into a communal fund on a weekly basis, and upon the death of a member, the fund would cover the expenses of rites and burial. These societies sometimes sold shares in the building of Columbaria, or burial vaults, owned by the fund. Other early examples of mutual surety and assurance pacts can be traced back to various forms of fellowship within the Saxon clans of England and their Germanic forebears, and to Celtic society.

Mathematical Formulae.

Foundation of modern actuarial calculation:

During the 17th century, a more scientific basis for risk management was being developed. In 1662, a London draper named John Graunt showed that there were predictable patterns of longevity and death in a defined group, or cohort, of people, despite the uncertainty about the future longevity or mortality of any one individual. This study became the basis for the original life table. Combining this idea with that of compound interest and annuity valuation, it became possible to set up an insurance scheme to provide life insurance or pensions for a group of people, and to calculate with some degree of accuracy each member's necessary contributions to a common fund, assuming a fixed rate of interest. The first person to correctly calculate these values was Edmond Halley.In his work, Halley demonstrated a method of using his life table to calculate the premium someone of a given age should pay to purchase a life-annuity.

Blaise Pascal & Probability Theory:

Pascal's development of probability theory was his most influential contribution to mathematics. Originally applied to gambling, today it is extremely important in economics, especially in actuarial science. John Ross writes, "Probability theory and the discoveries following it changed the way we regard uncertainty, risk, decision-making, and an individual's and society's ability to influence the course of future events." However, Pascal and Fermat, though doing important early work in probability theory, did not develop the field very far. Christiaan Huygens, learning of the subject from the correspondence of Pascal and Fermat, wrote the first book on the subject. Later figures who continued the development of the theory include Abraham de Moivre and Pierre-Simon Laplace.

In 1654, prompted by his friend the Chevalier de Méré, he corresponded with Pierre de Fermat on the subject of gambling problems, and from that collaboration was born the mathematical theory of probabilities. The specific problem was that of two players who want to finish a game early and, given the current circumstances of the game, want to divide the stakes fairly, based on the chance each has of winning the game from that point. From this discussion, the notion of expected value was introduced. Pascal later (in the *Pensées*) used a probabilistic argument, Pascal's wager, to justify belief in God and a virtuous life. The work done by Fermat and Pascal into the calculus of probabilities laid important groundwork for Leibniz' formulation of the calculus.

Life Expectancy:

Graunt's book *Natural and Political Observations Made upon the Bills of Mortality,* published in 1663, compiled and analysed data from the Bills of Mortality. Graunt, using the Rule of Three (mathematics) and ratios obtained by comparing years in the Bills of Mortality, was able to make estimates about the size of the population of London and England, birth rates and mortality rates of males and females, and the rise and spread of certain diseases.

Bills of Mortality:

Bill of Mortality from 1606, one of the earlier times which John Graunt looked at in his work. John Graunt's analysis in *Natural and Political Observations Made Upon the Bills of Mortality* consisted of a compilation and an analysis of data from the Bills of Mortality. The Bills of Mortality were documents offering information about the births, deaths, and causes of death in London parishes, printed and distributed weekly on Thursdays (in addition to an annual report released in December). The Bills of Mortality were said by Graunt to begin in 1592, and consistently released starting in 1603. Graunt describes how the data was collected for these Bills in his *Natural and Political Observations Made Upon the Mortality of Man*:

> "When anyone dies, then either by tolling, or by ringing of a Bell, or by bespeaking of a Grave of the Sexton, the same is known to the Searchers, corresponding with the said Sexton. The Searchers hereupon...examine by what Disease, or Casualty the corps died. Hereupon they make their Report to the Parish-Clerk, and he, every Tuesday night, carries in an Accompt of all the Burials, and Christnings, hapning that Week, to the Clerk of the Hall. On Wednesday the general Accompt is made up, and Printed, and on Thursdays published and dispersed to the several Families, who will pay for four shillings per Annum for them."

Graunt's description of the method of data collection for the Bills of Mortality also serves as an example of Graunt's use of scrutiny in appraising the data he was analysing. Graunt critiqued the collectors ("Searchers") who determined cause of death of the corpses; this critique manifested in Graunt's investigations into the effects on mortality of certain diseases, as Graunt suggested many causes of death were misrepresented.

Epidemiology:

Graunt's work reached rudimentary conclusions about the mortality and morbidity of certain diseases. Graunt was highly sceptical of the number of deaths recorded in the Bills of Mortality as due to the plague. Graunt speculated about the reasons for these miss-classifications, one of which includes the reliability of those reporting causes of death in the Bills of Mortality.

Another example of Graunt's work in epidemiology is his investigation of the sudden surge in deaths in 1634 due to Rickets. Graunt looked at two other causes of death--"Liver-grown" and "Spleen"--in addition to "Rickets," combining the three and comparing the frequency of deaths due to each cause between years. Graunt investigated if the sudden increase in deaths due to rickets in the Bills of Mortality was actually the result of misclassifying corpses who were said to have died from "Liver-grown" and "Spleen." Graunt concluded that "Rickets" as a cause of death was at a maximum for the first time.

Although the majority of his work was in Astronomy, Edmund Halley in 1693 published an article on life annuities, which featured an analysis of age-at-death on the basis of the Breslau statistics Caspar Neumann had been able to provide (Neumann's work has since been lost). This article allowed the British government to sell life annuities at an appropriate price based on the age of the purchaser. Halley's work strongly influenced the development of actuarial science (the discipline that applies mathematical and statistical methods to assess risk in insurance, finance, and other industries and professions). The construction of the life-table for Breslau, which followed more primitive work by John Graunt, is now seen as a major event in the history of demography.

Bernoulli Number:

The Swiss mathematician Jakob Bernoulli (1654–1705) was the first to realize the existence of a single sequence of constants B_0, B_1, B_2,… which provide a uniform formula for all sums of powers.

Bernoulli's formula for sums of powers is the most useful and generalizable formulation to date. The coefficients in Bernoulli's formula are now called Bernoulli numbers, following a suggestion of Abraham de Moivre.

Bayes Theorem:

In probability theory and statistics, **Bayes's theorem** (alternatively **Bayes's law** or **Bayes's rule**), named after Reverend Thomas Bayes, describes the probability of an event, based on prior knowledge of conditions that might be related to the event. For example, if the risk of developing health problems is known to increase with age, Bayes's theorem allows the risk to an individual of a known age to be assessed more accurately (by conditioning it on his age) than simply assuming that the individual is typical of the population as a whole.

Scottish Widows:

In March 1812, a number of prominent Scotsmen gathered in the Royal Exchange Coffee Rooms in Edinburgh. They were there to discuss setting up 'a general fund for securing provisions to widows, sisters and other female relatives' of fundholders so that they would not be plunged into poverty on the death of the fundholder during and after the Napoleonic Wars. Scottish Widows' Fund and Life Assurance Society opened in 1815 as Scotland's first mutual life office.

Its most noteworthy leader was the Very Rev. James Grant who served as its Director for 50 years (1840-1890). Scottish Widows used works by the following in putting its Fund together.

Dr. Robert Wallace (1697-1771), Professor and also Presbyterian preacher. “Dissertation on the Numbers of Mankind”

Dr. Alexander Webster (1707-1784), Church of Scotland Minister.

He propounded a scheme in 1742 for providing pensions for the widows of ministers. The tables which he drew up from information obtained from all the presbyteries of Scotland were based on a system of actuarial calculation that supplied a precedent followed by insurance companies in modern times for reckoning averages of longevity.

Webster published in 1748 his *Calculations*, setting forth the principles on which his scheme for widows' pensions was based; he also wrote a defence of the Methodist movement in 1742, and *Zeal for the Civil and Religious Interests of Mankind Commended* (1754).

In 1755 the government commissioned Webster to obtain data for the first census of Scotland, which he carried out in the same year. In 1753 he was elected moderator of the General Assembly; in 1771 he was appointed a dean of the Chapel Royal and chaplain to George III in Scotland.

Colin Maclaurin (1698-1746). Child Prodigy, M.A. At 14, world's youngest professor at 19 (record held till 2008) at Aberdeen University.

Maclaurin used Taylor series to characterize maxima, minima, and points of inflection for infinitely differentiable functions in his *Treatise of Fluxions*. Maclaurin attributed the series to Brook Taylor, though the series was known before to Newton and Gregory, and in special cases to Madhava of Sangamagrama in Fourteenth Century India. Nevertheless, Maclaurin received credit for his use of the series, and the Taylor series expanded around 0 is sometimes known as the *Maclaurin series*.

Maclaurin also made significant contributions to the gravitation attraction of ellipsoids, a subject that furthermore attracted the attention of d'Alembert, A.-C. Clairaut, Euler, Laplace, Legendre, Poisson and Gauss. Maclaurin showed that an oblate spheroid was a possible equilibrium in Newton's theory of gravity. The subject continues to be of scientific interest, and Nobel Laureate Subramanyan Chandrasekhar dedicated a chapter of his book *Ellipsoidal Figures of Equilibrium* to Maclaurin spheroids.

Independently from Euler and using the same methods, Maclaurin discovered the Euler–Maclaurin formula. He used it to sum powers of arithmetic progressions, derive Stirling's formula, and to derive the Newton-Cotes numerical integration formulas which includes Simpson's rule as a special case.

Maclaurin contributed to the study of elliptic integrals, reducing many intractable integrals to problems of finding arcs for hyperbolas. His work was continued by d'Alembert and Euler, who gave a more concise approach.

In his *Treatise of Algebra* (Ch. XII, Sect 86), published in 1748 two years after his death, Maclaurin proved a rule for solving square linear systems in the cases of 2 and 3 unknowns, and discussed the case of 4 unknowns. This publication preceded by two years Cramer's publication of a generalization of the rule to *n* unknowns, now commonly known as Cramer's rule.