Fibonacci's Mathematical Contributions

Introducing the Decimal Number system into Europe

He was one of the first people to introduce the Hindu-Arabic number system into Europe - the positional system we use today - based on ten digits with its decimal point and a symbol for zero:

1 2 3 4 5 6 7 8 9 0

His book on how to do arithmetic in the decimal system, called **Liber abbaci** (meaning *Book of the Abacus* or *Book of Calculating*) completed in 1202 persuaded many European mathematicians of his day to use this "new" system.

The book describes (in Latin) the rules we all now learn at elementary school for adding numbers, subtracting, multiplying and dividing, together with many problems to illustrate the methods:

```
1 7 4 +     1 7 4 -      1 7 4 x     1 7 4 ÷ 28
  2 8         2 8          2 8             is
-----       -----      -------
2 0 2       1 4 6      3 4 8 0 +     6 remainder 6
-----       -----      1 3 9 2
                       -------
                       4 8 7 2
                       -------
```

Let's first of all look at the Roman number system still in use in Europe at that time (1200) and see how awkward it was for arithmetic.

Roman Numerals

The Numerals are letters

The method in use in Europe until then used the Roman numerals:

```
I = 1,
V = 5,
X = 10,
L = 50,
C = 100,
D = 500 and
M = 1000
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You can still see them used on foundation stones of old buildings and on some clocks.

The Additive rule

The simplest system would be merely to use the letters for the values as in the table above, and add the values for each letter used.
For instance, 13 could be written as XIII or perhaps IIIX or even IIXI. This occurs in the Roman language of Latin where 23 is spoken as *tres et viginti* which translates as *three and twenty*. You may remember the nursery rhyme **Sing a Song of Sixpence** which begins

Sing a song of sixpence
A pocket full of rye
Four and twenty blackbirds
Baked in a pie...

Above 100, the Latin words use the same order as we do in English, so that whereas 35 is *quinque et triginta* (5 and 30), 235 is *ducenti triginta quinque* (two hundred thirty five).

In this simple system, using addition only, 99 would be 90+9 or, using only the numbers above, 50+10+10+10 + 5+1+1+1+1 which translates to LXXXXVIIII and by the same method 1998 would be written by the Romans as MDCCCCLXXXXVIII.
But some numbers are long and it is this is where, if we agree to *let the order of letters matter* we can also use *subtraction*.

The subtractive rule

The Roman language (Latin) also uses a subtraction principle so that whereas 20 is *viginti* 19 is "1 from 20" or *undeviginti*. We have it in English when we say the time is "10 to 7" which is not the same as "7 10". The first means 10 minutes before (or subtracted from) 7 0'clock, whereas the second means 10 minutes added to (or after) 7 o'clock. This is also reflected in Roman numerals.
This abbreviation makes the *order* of letters important. So if a smaller value came *before* the next larger one, it was *subtracted* and if it came after, it was *added*.
For example, XI means 10+1=11 (since the smaller **one** comes after the larger **ten**) but IX means 1 less than 10 or 9.

But 8 is still written as VIII (not IIX). The subtraction in numbers was only of a unit (1, 10 or 100) taken away from 5 of those units (5, 50 or 500 or from the next larger multiple of 10 (10, 100 or 1000).

Using this method, 1998 would be written much more compactly as MCMXCVIII but this takes a little more time to interpret: 1000 + (100 less than 1000) + (10 less than 100) + 5 + 1 + 1 + 1.
Note that in the UK we use a similar system for time when 6:50 is often said as "ten to 7" as well as "6 fifty", similarly for "a quarter to 4" meaning 3:45. In the USA, 6:50 is sometimes spoken as "10 of 7".

> *Look out for Roman numerals used as the date a film was made, often recorded on the screen which gives its censor certification or perhaps the very last image of the movie giving credits or copyright information.*

Arithmetic with Roman Numerals

Arithmetic was not easy in the Roman system:

```
CLXXIIII added to XXVIII  is CCII
CLXXIIII less     XXVIII  is CXXXXVI
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For more on Roman Numerals, see the excellent [Frequently Asked Questions on Roman Numerals] at Math Forum.

The Decimal Positional System

The system that Fibonacci introduced into Europe came from India and Arabia and used the Arabic symbols 1, 2, 3, 4, 5, 6, 7, 8, 9 with, most importantly, a symbol for zero 0.
With Roman numbers, 2003 could be written as MMIII or, just as clearly, it could be written as IIIMM - the order does not matter since the values of the letters are added to make the number in the original (unabbreviated) system. With the abbreviated system of IX meaning 9, then the order did matter but it seems this sytem was not often used in Roman times.
In the "new system", the order *does* matter *always* since 23 is quite a different number to 32. Also, since the *position* of each digit is important, then we may need a *zero* to get the digits into their

correct places (columns) eg 2003 which has no tens and no hundreds. (The Roman system would have just omitted the values not used so had no need of "zero".)

This *decimal positional system*, as we call it, uses the ten symbols of Arabic origin and the "methods" used by Indian Hindu mathematicians many years before they were imported into Europe. It has been commented that in India, the concept of *nothing* is important in its early religion and philosophy and so it was much more natural to have a symbol for it than for the Latin (Roman) and Greek systems.

"Algorithm"

Earlier the Persian author Abu ‘Abd Allah, Mohammed ibn Musa al-Khwarizmi (usually abbreviated to Al-Khwarizmi had written a book which included the rules of arithmetic for the decimal number system we now use, called *Kitab al jabr wa‘l-muqabala* (Rules of restoring and equating) dating from about 825 AD. D E Knuth (in the errata for the second edition and third edition of his "Fundamental Algorithms") gives the full name above and says it can be translated as *Father of Abdullah, Mohammed, son of Moses, native of Khwarizm*. He was an astromomer to the caliph at Baghdad (now in Iraq).

- Al-Khowârizmî is the region south and to the east of the Aral Sea around the town now called **Khiva** (or Urgench) on the Amu Darya river. It was part of the Silk Route, a major trading pathway between the East and Europe. In 1200 it was in Persia but today is in Uzbekistan, part of the former USSR, north of Iran, which gained its independence in 1991.
- Prof Don Knuth has a picture of a postage stamp issued by the USSR in 1983 to commemorate al-Khowârizmî 1200 year anniversary of his probable birth date.
- From the title of this book *Kitab* ***al jabr*** *w'al-muqabala* we derive our modern word **algebra**.
- The Persian author's name is commemorated in the word **algorithm**. It has changed over the years from an original European pronunciation and latinisation of *algorism*. Algorithms were known of before Al-Khowârizmî's writings, (for example, Euclid's *Elements* is full of algorithms for geometry, including one to find the greatest common divisor of two numbers called *Euclid's algorithm* today).
- The USA Library of Congress has a list of citations of Al-Khowârizmî and his works.

Our modern word "algorithm" does not just apply to the rules of arithmetic but means ***any precise set of instructions for performing a computation*** whether this be
- a method followed by humans, for example:
    - a cooking **recipe**;
    - a knitting **pattern**;
    - travel **instructions**;
    - a car **manual page** for example, on how to remove the gear-box;
    - a medical **procedure** such as removing your appendix;
    - a calculation by **human computors** : two examples are:
        - William Shanks who computed the value of pi to 707 decimal places by hand last century over about 20 years up to 1873 - but he was wrong at the 526-th place when it was checked by desk calculators in 1944!
        - Earlier Johann Dase had computed pi correctly to 205 decimal places in 1844 when aged 20 but *this was done completely in his head* just writing the number down after working on it for two months!!
- or **mechanically** by machines (such as placing chips and components at correct places on a circuit

board to go inside your TV)
- or **automatically** by electronic computers which store the instructions as well as data to work on.

See D E Knuth, The Art of Computer Programming *Volume 1: Fundamental Algorithms* (now in its Third Edition, 1997)pages 1-2.
There is an English translation of the ".. al jabr .." book: L C Karpinski **Robert of Chester's Latin Translation ... of al-Khowarizmi** published in New York in 1915. [Note the variation in the spelling of "Al-Khowârizmî" here - this is not unusual! Other spellings include al-Khorezmi.]
Ian Stewart's **The Problems of Mathematics** (Oxford) 1992, ISBN: 0-19-286148-4 has a chapter on algorithms and the history of the name: *chapter 21: Dixit Algorizmi*.

The Fibonacci Numbers

In Fibonacci's *Liber Abaci* book, chapter 12, he introduces the following problem (here in Sigler's translation - see below):

> **How Many Pairs of Rabbits Are Created by One Pair in One Year**
> A certain man had one pair of rabbits together in a certain enclosed place, and one wishes to know how many are created from the pair in one year when it is the nature of them in a single month to bear another pair, and in the second month those born to bear also.

He then goes on to solve and explain the solution:

> Because the above written pair in the first month bore, you will double it; there will be two pairs in one month.
> One of these, namely the first, bears in the second montth, and thus there are in the second month 3 pairs;
> of these in one month two are pregnant and in the third month 2 pairs of rabbits are born, and thus there are 5 pairs in the month;
> ...
> there will be 144 pairs in this [the tenth] month;
> to these are added again the 89 pairs that are born in the eleventh month; there will be 233 pairs in this month.
> To these are still added the 144 pairs that are born in the last month; there will be 377 pairs, and this many pairs are produced from the abovewritten pair in the mentioned place at the end of the one year.
>
> You can indeed see in the margin how we operated, namely that we added the first number to the second, namely the 1 to the 2, and the second to the third, and the third to the fourth and the fourth to the fifth, and thus one after another until we added the tenth to the eleventh, namely the 144 to the 233, and we had the abovewritten sum of rabbits, namely 377, and thus you can in order find it for an unending number of months.

beginning 1
first 2
second 3
third 5
fourth 8
fifth 13
sixth 21
seventh 34
eighth 55
ninth 89
tenth 144
eleventh 233
end 377

Did Fibonacci invent this Series?

Fibonacci says his book *Liber Abaci* (the first edition was dated 1202) that he had studied the "nine Indian figures" and their arithmetic as used in various countries around the Mediterranean and wrote about them to make their use more commonly understood in his native Italy. So he probably merely included the "rabbit problem" from one of his contacts and did not invent either the problem

or the series of numbers which now bear his name.
D E Knuth adds the following in his monumental work *The Art of Computer Programming: Volume 1: Fundamental Algorithms* errata to second edition:

> Before Fibonacci wrote his work, the sequence **F(n)** had already been discussed by Indian scholars, who had long been interested in rhythmic patterns that are formed from one-beat and two-beat notes. The number of such rhythms having **n** beats altogether is **F(n+1)**; therefore both Gospala (before 1135) and Hemachandra (c. 1150) mentioned the numbers 1, 2, 3, 5, 8, 13, 21, ... explicitly.